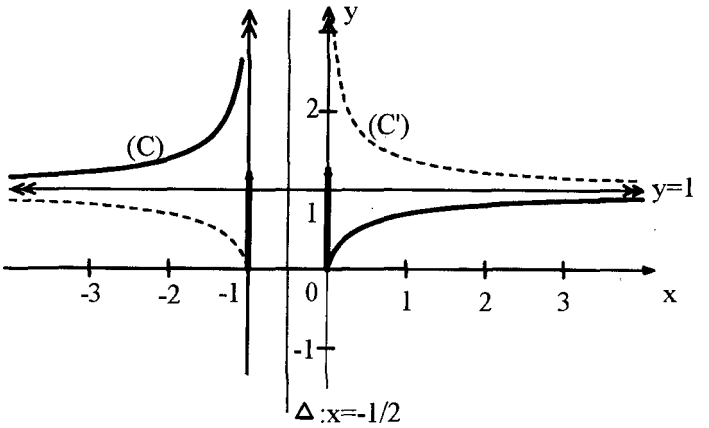
x	$-\infty$		-1	∴∴∴	0		$+\infty$
f'(x)	+			∴∴∴	+		
f(x)	1	↗	$+\infty$	∴∴∴	0	↗	1

- $\lim\limits_{-\infty} f = \lim\limits_{x\to-\infty} x\text{Log}\left(1+\frac{1}{x}\right) = \lim\limits_{X\to0^-} \frac{\text{Log}(1+X)}{X} = 1$ (avec $X=\frac{1}{x}$)
- $\lim\limits_{+\infty} f = \lim\limits_{x\to+\infty} x\text{Log}\left(1+\frac{1}{x}\right) = \lim\limits_{X\to0^+} \frac{\text{Log}(1+X)}{X} = 1$ (avec $X=\frac{1}{x}$)
- $\lim\limits_{-1^-} f = \lim\limits_{x\to-1^-} \left[x\text{Log}\left(1+\frac{1}{x}\right)\right] = +\infty$
- $\lim\limits_{x\to\pm\infty} f(x) = 1 \Rightarrow$ (C) possède la droite d'équation y = 1 comme asymptote horizontale au voisinage de $\pm\infty$.



3/a- $\mathscr{A}(\alpha) = \int_\alpha^1 f(x)dx = \int_\alpha^1 x\text{Log}\left(1+\frac{1}{x}\right)dx$

$$\left(\begin{array}{lll} u'(x) = x & \nrightarrow & u(x) = \frac{1}{2}x^2 \\ v(x) = \text{Log}(1+\frac{1}{x}) & \nrightarrow & v'(x) = \frac{-1}{x(x+1)} \end{array}\right)$$

$$= \left[\frac{1}{2}x^2\text{Log}\left(1+\frac{1}{x}\right)\right]_\alpha^1 + \frac{1}{2}\int_\alpha^1 \frac{x}{x+1}dx$$

$$= \frac{1}{2}\text{Log}2 - \frac{1}{2}\alpha^2\text{Log}\left(1+\frac{1}{\alpha}\right) + \frac{1}{2}\int_\alpha^1 \frac{x+1-1}{x+1}dx$$

$$= \frac{1}{2}\text{Log}2 - \frac{1}{2}\alpha^2\text{Log}\left(1+\frac{1}{\alpha}\right) + \frac{1}{2}[x-\text{Log}(x+1)]_\alpha^1$$

$$= \frac{1}{2} - \frac{1}{2}\alpha^2\text{Log}\left(1+\frac{1}{\alpha}\right) - \frac{1}{2}\alpha + \frac{1}{2}\text{Log}(\alpha+1)$$